

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)**SciVerse ScienceDirect**

Procedia - Social and Behavioral Sciences 54 (2012) 1270 – 1277

---

---

**Procedia**  
Social and Behavioral Sciences

---

---

EWGT 2012

15th meeting of the EURO Working Group on Transportation

# A global optimization approach to solve the traffic signal synchronization problem

L. Adacher<sup>\*</sup>*Roma Tre University, Via della Vasca Navale 79, Roma 00146, Italy*

---

## Abstract

The Traffic Signal Synchronization is a traffic engineering technique of matching the green light times for a series of intersections to enable the maximum number of vehicles to pass through, thereby reducing stops and delays experienced by motorists. Synchronizing traffic signals ensures a better flow of traffic and minimizes gas consumption and pollutant emissions. The objective function used in this work is a weighted sum of the delays caused by the signalized intersections. In this paper, we apply generalized 'surrogate problem' methodology that is based on an on-line control scheme which transforms the problem into a 'surrogate' continuous optimization problem and proceeds to solve the latter using standard gradient-based approaches while simultaneously updating both actual and surrogate system states. We extend a 'surrogate problem' approach that is developed for a class of stochastic discrete optimization problems so as to tackle the traffic signal synchronization problem to minimize the total delay (DTSS). Numerical experiments conducted on a test and a real networks show that the surrogate method converges in a very small area.

© 2012 Published by Elsevier Ltd. Selection and/or peer-review under responsibility of the Program Committee

Open access under [CC BY-NC-ND license](https://creativecommons.org/licenses/by-nc-nd/4.0/).

**Keywords:** Stochastic Optimization, Surrogate Method, Traffic Signal Synchronization.

---

## 1. Introduction

The performance of a traffic network can be influenced through several types of actions or decision variables. Some of these pertain to changing the load pattern on the network, through demand management actions, including attempts to route vehicles optimally through the network; others pertain to how traffic flow is controlled through the network components, such as junction utilization through signal control (supply management). Although the potential of explicitly combining both types of actions, especially joint signal control

---

<sup>\*</sup> Corresponding author. Tel.: +39-06-57333318; fax: +39-06-57333612.

E-mail address: [adacher@dia.uniroma3.it](mailto:adacher@dia.uniroma3.it)

and route assignment, long has been suggested, most of the research and virtually all the approaches used in practice have followed one of two schemes:

- (a) considering the signal control to be fixed and using traffic assignment as decision variables (traffic assignment models), and
- (b) considering traffic assignment (loading pattern) to be fixed and using signal control as decision variables (signal optimization models).

Nonetheless, several researchers have recognized the interaction between signal control and traffic assignment.

Usual traffic signal optimization methods seek either to maximize the green bandwidth or to minimize a general objective function that typically includes delays, number of stops, fuel consumptions and some external costs like pollutant emissions. Without loss of generality, we refer to this method in the following as minimum delay problem. This method is related to physical variables that are to be minimized; anyway, it is a non-convex problem and existing solution methods do not guarantee to achieve the optimal solution. The maximal bandwidth method maximizes an opportunity of progression for drivers and does not reduce delays necessarily; nevertheless, it is a quasi-concave problem and efficient solving algorithms exist to find the optimal solution.

Traffic signal optimization on road arteries consists of two problems: the solution algorithm and the progression model used to compute the values of the objective function.

In order to improve the algorithm, several authors combined in a different way the two synchronization approaches, that is the minimum delay and the maximal bandwidth. Cohen [1] used the maximal bandwidth as initial solution of the former problem; Cohen and Liu [2] constrained the solution of the former problem to fulfil maximum bandwidth; Hadi and Wallace [3] used the bandwidth as objective function; Malakapalli and Messer [4] added a simple delay model to the maximal bandwidth algorithm; Gartner and Hou [5] introduced a flow-dependent bandwidth function; Papola and Fusco [6] and Adacher and Cipriani [7] have expressed the delay at nodes as a closed form function of the maximal bandwidth solution. Since the first platoon dispersion model introduced by Robertson [8], progressively more complex models have been developed. Park [9] introduced a genetic algorithm based traffic signal optimization program for oversaturated intersections consisting of two modules: a genetic algorithm optimizer and microscopic simulator. Dazhi [10] proposed a bi-level programming formulation and a heuristic solution approach for dynamic traffic signal optimization in networks with time dependent demand and stochastic route choice. Chang and Sun [11] proposed a dynamic method to control an oversaturated traffic signal network by utilizing a bang-bang-like model for oversaturated intersections and TRANSYT for the unsaturated intersections. The TRANSYT traffic model [12] simulates the movement of traffic through a network and takes into account of the platoon dispersion effect. It is a widely used procedure to determine the queues and delays in a signal-controlled network with explicit consideration of the signal coordination effects. However, the traffic model does not consider the re-routing of traffic in the network in response to the traffic conditions. In this paper, the TRANSYT traffic model is employed for the evaluation of delays in the network, which forms the basic module of the problem in the paper.

We have utilized the platoon based delay model [13], it is possible to deal with even non stationary traffic demand and non synchronized signal settings. The model is rather similar the well-established TRANSYT solving procedure [14], which respects to it introduces some additional flexibility aimed at improving the algorithm efficiency.

## 2. Delay Model

The traffic model provides the queue length and the platoons average delay, see Colombaroni et al. [13] for more details.

The delay caused by a signalized intersection is strictly defined as the difference in the road section travel time in the presence of traffic lights compared to the travel time of the same section if a vehicle could travel along a

trajectory at constant speed  $v$  (*speed of synchronization*). Compared to traditional delay formulations, this model includes both the upstream and the downstream node delay.

This model is based on the platoons representation and it simulates congested road with propagation of the queue between nodes (spill-back phenomena). The delay computation requires an iterative procedure that classifies the different platoons progressively. It is worth noting that such a procedure involves few iterations, because the platoons can both catch up each other along the links and recombine themselves at nodes, when more platoons arrive during the red phase.

The simulation uses three basic laws:

- **Departure law**: to generate the initial platoons and the new platoons during the simulation;
- **Arrivals law**: to simulate the progression and recombination of platoons;
- **Classification law**: to calculate the platoons delay.

Each of these laws, suitably iterated, estimates the total delay.

The objective function is a linear combination of the total delay on each direction of the artery and it is calculated via simulation.

$$J = (1 - \omega_l) \left( \omega_l \sum_{i=1}^n \omega_i D_i^{(a)} + (1 - \omega_l) \sum_{i=1}^n \omega_i D_i^{(-a)} \right) + \omega_l \sum_{i=1}^n \omega_i D_{ih}^{(t)} = J(D) \quad (1)$$

- $D_i^{(a)}$ : the total delay at node  $i$  in one direction of the artery  $a$
- $D_i^{(-a)}$ : the total delay at node  $i$  in the opposite way of the artery  $a$
- $D_{ih}$ : the total delay at node  $i$  of queue  $h$  in lateral approach  $t$
- $\omega_a$ : the weight of delay in direction  $a$
- $\omega_l$ : the weight of the delay at lateral approaches
- $\omega_i$ : the weight of node  $i$

Analyzing a road artery, the problem of minimum travel time or minimum delay for traffic signals synchronization (DTSS) can be expressed as follows:

$$\min J(D) = \min f(C, g; \vartheta, L, s.X, Q) \quad (2)$$

subject to:

- $0 \leq \theta_i < C_i$
- $C_{min} \leq C_i \leq C_{max}$
- $\max\{\gamma_{i,a} \cdot C_i\} \leq g_i \leq C_i - L_i - \max\{\gamma_{i,t} \cdot C_i\}$

where:

- $C_i$ : the traffic light cycle for the intersection  $i$ . The traffic light cycle is defined as any complete sequence of switch on (and off) of traffic lights at the end of which returns the same configuration of the lights existing at the beginning of the sequence.
- $\mu_i$ : the offsets for the intersection  $i$ .
- $g_i$ : is the effective green time of node  $i$ ;
- $L_i$ : the time loss at the node  $i$ . That is the time in which the intersection is not completely used. The time lost is mainly due to three contributions:
  - Transient state of vehicles in the queue at the beginning of the green phase;
  - Transient state of exiting vehicles at the end of the green phase and during the yellow phase;
  - The time between the end of yellow and the beginning of green of the next phase.

The lost times at the beginning and at the end of green are used to determinate the duration of effective green.

- $\gamma_{i,a}$ ,  $\gamma_{i,t}$ : saturation degree of the approach  $a$  along the artery and of the transversal approach  $t$  of the  $i$  node. The saturation degree is the ratio between traffic flow and the saturation flow. This quantity is an indicator of the level of congestion.
- $s$ : saturation flow vector for each arc. The saturation flow is the maximum number of vehicles that can cross a stop intersection line per unit time, in the presence of continuous queue. The saturation flow depends on the geometric characteristics of the intersection, on flow composition and on the control traffic lights.
- $X$ : urban artery geometry.
- $Q$ : demand level or vehicular flow. It defines the flow of a current the average number of vehicles passing through a section in unit time.

The travel time of a road section is closely linked to the geometry of the road itself and to the configuration of traffic light plans. Saturation flow and geometry are studied and designed in earlier phases and often they cannot be changed a posteriori. In this work that are parameters. In addition, to maximize the intersection performance, time loss are already designed to be both minimum and ensure the safety levels required.

Among the variables that have been used to express the system are only three: *cycles*, *green splits* and *offsets* are our state variables.

### 3. Analysis of objective function

The DTSS problem can be formulated as the problem of finding out the cycle  $C^*i$  (we have considered the same cycle for all junctions), the vector of green split ratios  $g^*$  and the vector of offsets  $\mu^*$ , that minimize an objective function  $J(D)$ , which expresses the total delay time on the network. Numerical experiments have been conducted on four test networks considering different dimensions and characteristics, all instances are major urban roads and were simulated during the peak hour.

The objective function analysis puts in evidence that the shape of the objective function is quasi-convexity related to the cycle and related to the green vector is evident a large quasi-convexity shape.

The real problem is the offset, the shape presents many local minima; in Fig.1 the shape related to the green time variation and the offset variation is depicted.

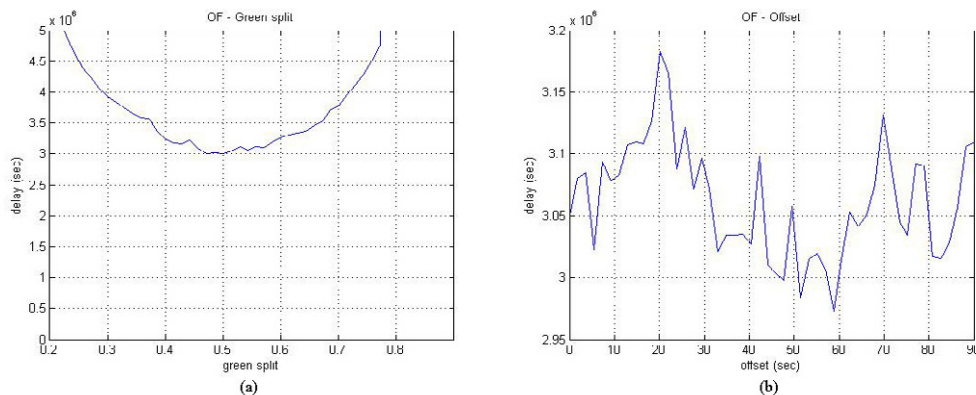


Figure 1. Analysis of the solution space varying: a) a component of green split vector, b) a component of the offset vector

To visualize the characteristics of the solution space it is not necessary to plot the average system time for all possible combinations of cycle/green time ratio/offset. To show the characteristics of the solution space, the cycle, the vector of the offset and all green time ratio are kept constants, except for one component. It is done for

different values of green/offset/cycle that are kept constants. It is evident that to find an optimal solution for the cycle variable it is not needed a complicated method, for these reasons we have decided to investigate the cycle by binary research. For the other two variables we have try to apply the surrogate method.

#### 4. The Surrogate Method (SM) applied to DSST problem

In this paper we proposed an approach for solving such problems based on the idea of transforming a *discrete* optimization problem into a "surrogate" *continuous* optimization problem which is not only easier to solve, but also much faster using standard gradient-based approaches [19]. The two key issues related to this approach are (a) obtaining the actual solution of the original problem from the surrogate one, and (b) using this approach on-line, i.e., making sure that at every step of the iterative solution process a *feasible* discrete state is defined from an (infeasible) surrogate state. This has two advantages:

First, the cost of the original system is continuously adjusted (in contrast to an adjustment that would only be possible at the end of the surrogate minimization process);

and Second, it allows us to make use of information typically employed to obtain cost sensitivities from the *actual* operating system at every step of the process.

This scheme is intended to combine the advantages of a stochastic approximation type of algorithm with the ability to obtain sensitivity estimates with respect to discrete decision variables.

The DTSS problem can be formulated as the problem of finding out the cycle  $C_i$  (we have considered the same cycle for all junctions), the vector of green split ratios  $g^*$  and the vector of offsets  $\theta^*$ , that minimize an objective function  $J(D)$ , which expresses the total delay time on the network. In this paper, we tackle the DTSS problem using an *on-line* optimization approach, i.e., for a fixed value of the cycle, we iteratively adjust the (integer) green split ratios  $g_1, \dots, g_n$  and the offsets  $\theta_1, \dots, \theta_n$  for  $n$  intersection links based on data directly observed and aiming at minimizing the global performances of the network, the overall *mean delay* (or delay) of cars, denoted by  $J(D) = J(C, g_1 \dots g_n, \theta_1 \dots \theta_n)$ . The cycle is analyzed by a binary search, for a given fixed value of the cycle  $C_i$ ,  $g$  and  $\theta$  are evaluated on the bases of the surrogate approach, we denote with  $x$  the state variable of the surrogate method.

For a fixed  $C_i$ , the DTSS problem can be formulated as follows:

$$\min_{(g, \theta) \in A_d} J_d(C_i, g, \theta) \quad (3)$$

where

- $C_i$  is the cycle fixed by the binary research
- $g$  is an  $n$ -dimensional decision vector with  $g_i \in \mathbb{Z}^+$  denoting the green time ratio for intersection link  $i$
- where  $\theta$  is an  $n$ -dimensional decision vector with  $\theta_i \in \mathbb{Z}^+$  denoting the offset for intersection link  $i$

the capacity constraint

$$A_d = \{g := [g_1, \dots, g_n], g_{\min} \leq g_i \leq g_{\max}, g_i \in \mathbb{Z}^+; \theta := [\theta_1, \dots, \theta_n] 0 \leq \theta_i \leq 1; \theta_i \in \mathbb{Z}^+\}$$

and  $J(C, g, \theta) = J(D)$ , that is the total travel time on the network when the variables (green split vector, offsets and cycle) are fixed.

We have considered different application of the surrogate method:

- **USM** (Unique Surrogate Method): the surrogate method is applied at whole system and a unique variable state  $x = (g; \theta)$  2n dimensional is considered and a unique dynamic step size is fixed
- **USMD** (Unique Surrogate Method and Different step size): the surrogate method is applied at whole system and a unique variable state  $x = (g; \theta)$  2n dimensional is considered and a different dynamic step size is fixed for  $g$  and  $\mu$  respectively
- **SMS** (Surrogate Method in Sequence): the surrogate method is applied two times in sequence, first  $x = \theta$  and when an optimal value of the offset is defined  $x = g$ .

All junctions are supposed having same cycle and the value of the cycle is calculated on binary search, fixed this value the state  $x$  is calculated by Surrogate Method. For the green split vector we have considered two types of constraints, one is based on safety rules the other is presented in the delay model section, it is possible that the combination of these two different constraints can reduce the admissible values for  $g$  ( $g_{min} < g_i < g_{max}$ ).

We can summarize the results of the surrogate method as the following optimization algorithm for the solution of the basic problem:

First, for a given  $C_i$ , initialize  $\rho_0 = x_0$  and perturb  $\rho_0$  to have all components non-integer. Then, for any iteration  $k = 0, 1, \dots$

- (1) Determine  $N(\rho_k)$ , the set of all feasible neighbourhood discrete states of  $\rho_k$

$$N(\rho_k) = \{x \mid x = \inf \rho_k + \bar{x}, \forall \bar{x} \in \{0, 1\}^N\} \cap A_d$$

- (2) Determine  $S(\rho_k)$ , a selection set to define a set whose a convex hull includes  $\rho_k$  [using ([19]) ].

- (3) Select a transformation function  $f_k \in F(\rho_k)$  such that

$$x_k = f_k(\rho_k) = \arg \min_{r \in S(\rho_k)} \|x - \rho_k\|$$

- (4) Evaluate the gradient estimation

$$\nabla L_c(\rho_k) = [\nabla_1 L_c(\rho_k), \dots, \nabla_N L_c(\rho_k)] \text{ using the following relationship}$$

$$\nabla_j L_c = L_d(x^j) - L_d(x^k) \text{ where } k \text{ satisfies } x^j - x^k = e_j$$

- (5) Update state:  $\rho_{k+1} = \pi_{k+1}[\rho_k - \eta_k \nabla L_c(\rho_k)]$

- (6) If some stopping condition is not satisfied, repeat steps for  $k+1$ . Else set  $\rho^*$

Finally, we obtain  $x^*$  as one of the neighbouring feasible states in the set  $S(\rho^*)$ . It is important to notice that to evaluate the gradient estimation, we need to calculate  $2 * |x|$  times the value of the object function, so this estimation is dependent of the decision vector dimension.

In step 5,  $\eta_k$  is the step size and  $\pi_{k+1}$  is a projection such that if  $\rho_{k+1} \in A_c$ , then  $\pi_{k+1}(\rho_{k+1}) = \rho_{k+1}$ , otherwise, the projection maps  $\rho_{k+1}$  to a point in  $A_c$  which is closest to  $\rho_{k+1}$ . The step size and the projection mapping are a crucial element and may have a significant effect on convergence, we have fixed two different step size related to the two components of the state  $(g; \mu)$ . When we consider just one state variable the better step size is  $1/\nabla L(\rho)$ , if we consider two different step size for the green vector the best step size is the same but for the offset is  $n/\nabla L(\rho)$ .

For the green split vector the SM converges also with a static step and for the offset vector we have fixed a dynamic step, but fast convergence is found by dynamic step.

## 5. Numerical Results

Numerical experiments have been conducted on four real test networks considering different dimensions and characteristics as shown in table 1. All instances are major urban roads and were simulated during the peak hour.

All the algorithms give a very small area of convergence, the better solution is given by SMS with a gap of 3 % compared to best solution. In terms of efficiency, the SMS gives better solution and just in some cases the algorithms find almost the same solution, see Fig. 2. The SMS gives a reduction of the total delay between 10%

and 45% compared to the others two. In terms efficacy the *SMS* gives always worst performances because the Surrogate methods in this case is applied two times, so in the worst case it needs the double evaluation of the fitness.

Table 1. Characteristics of instances

Signals	Type	Capacity	Flow	Cross Flow	Way
4	ring road	high	high	high	two
4+1 pedes.	radial road	high	high	low	two
5	ring road	high	high	high	one
8	radial road	very high	very high	low	two

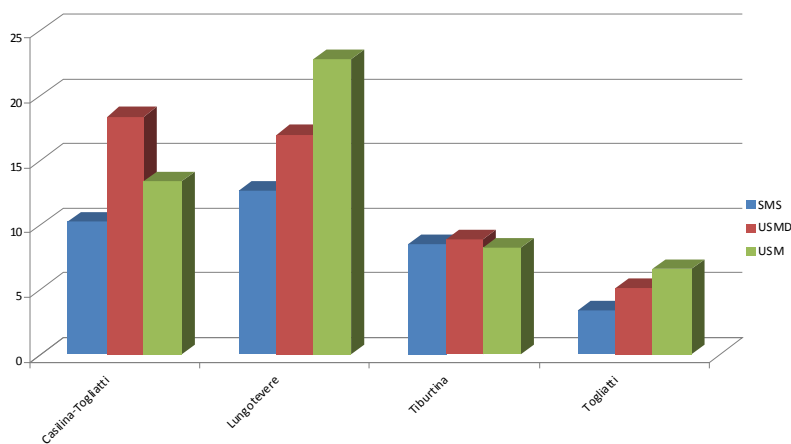


Figure 2. Objective function comparison

## 6. Conclusions

In this paper, we discuss different procedures for solving the Traffic Signal Synchronization problem to minimize the total Delay (DTSS). We apply the generalized 'surrogate problem' methodology that is based on an on-line control scheme which transforms the problem into a 'surrogate' continuous optimization problem and proceeds to solve the latter using standard gradient-based approaches while simultaneously updating both actual and surrogate system states. We have tested this method on some test networks with different parameters and we have compared the different approaches on real networks. The results put in evidence that for this type of problem the best results is given by *SMS* but obviously *UMS* it always implies an efficiency improvement.

## References

- Cohen S.L., Concurrent Use of MAXBAND and TRANSYT Signal Timing Programs for Arterial Signal Optimization, Transportation Research Record, Vol. 906, pp. 81-84, 1983.
- Cohen S. and Liu C., The Bandwidth Constrained TRANSYT Signal-Optimization Program Transportation Research Record, Vol. 1057, pp.1-7, 1986.
- Hadi M. and Wallace C., Algorithm to Optimize Signal Phasing and Transportation Research Record, Vol. 1421, 1993.

- Malakapalli M.P. e Messer C.J., Enhancements to the PASSER II-90 Delay Estimation Procedures, Transportation Research Record, Vol. 1421, pp. 94-103, 1983.
- Gartner N. and Hou G., Performance Evaluation of Multi-Band Progression Method, 7<sup>th</sup> IFAC/IFORS Symposium on "Transportation Systems: Theory and Application of Advanced Technology", Tianjin, China, pp. 981-986, 24-26 August, 1994.
- Adacher L. and Cipriani E., A surrogate approach for the global optimization of signal settings and traffic assignment problem, ITSC Madeira Island, Portugal, 19-22 September 2010.
- Papola N. and Fusco G., A new analytical model for traffic signal synchronization, Proc. Of the 2nd ICTTS Conference, Beijing, China, 31 July-2 August, 2000.
- Robertson D., Transyt: a Traffic Network Study Tool Ministry of Transport, RRL Report LR253, Crowthorne, Berkshire, United Kingdom, 1969.
- Park B., Messer C. and Urbanik-II T., Traffic Signal Optimization for Oversaturated Conditions: Genetic Algorithm Approach, Transportation Research Record, Vol. 1683, pp. 133-142, 1999.
- Dazhi S., Benekohal R.F. and Waller S.T., Bilevel Programming Formulation and Heuristic Solution Approach for Dynamic Traffic Signal Optimization, Computer-Aided Civil and Infrastructure Engineering, 21, pp. 321-333, 2006.
- Chang T. and Sung, G., Modeling and Optimization of an Oversaturated Signalized Network, Transportation Research - Part B, Vol. 38, pp. 687-707, 2004.
- Vincent R.A., Mitchell A.I., Robertson D.I., User guide to TRANSYT version 8. Report LR888., Transport and Road Research Laboratory, Department of the Environment, Department of Transport, Crowthorne, Berkshire, 1980.
- Colombaroni C., Fusco G. and Gemma A., Joint Problem of Traffic Signal Synchronization and Bus Priority, Seventh Triennial Symposium on Transportation Analysis. Tristan VII, Tromso, 20-25 June 2010.
- McTrans-Center Traffic Network Study Tool - Transyt7F. United States Version, 2006.
- Holland J.H., Adaptation in natural and artificial systems, MIT Press, 1992
- Colombaroni C., Fusco G. and Gemma A., Optimization of Traffic Signals on Urban Arteries through a Platoon-Based Simulation Model, 11th Int. Conf. on Automatic Control, Modelling and Simulation, Istanbul, Turkey, May-June 2009
- Webster F. V., Traffic signal settings, Road Research technical Paper No.39, HMSO, London, 1958.
- Gokbayrak K. and Cassandras C. G., A Generalized Surrogate Problem Methodology for On-Line Stochastic Discrete Optimization, J. of Optimization Theory and Applic., Vol. 114, 1, pp. 97-132, 2002.
- Gokbayrak K. and Cassandras C. G., An On-Line 'Surrogate Problem' Methodology For Stochastic Discrete Resource Allocation Problems, J. of Optimization Theory and Applications, Vol. 108, 2, pp. 349-376, 2001.